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ANALYSIS OF AN RNG BASED TURBULENCE MODEL FOR SEPARATED FLOWS*

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ABSTRACT

A two-equation turbulence model of the $K-\varepsilon$ type was recently derived by Yakhot & Orszag based on Renormalization Group (RNG) methods. It was later reported that this RNG based model yields substantially better predictions than the standard $K-\varepsilon$ model for turbulent flow over a backward facing step - a standard test case used to benchmark the performance of turbulence models in separated flows. The apparent improvements obtained from the RNG $K-\varepsilon$ model were attributed to the better treatment of near wall turbulence effects. In contrast to these earlier claims, it is shown in this paper that the original version of the RNG $K-\varepsilon$ model substantially underpredicts the reattachment point in the backstep problem - a deficiency that is traced to the modeling of the production of dissipation term. However, with the most recent improvements in the RNG $K-\varepsilon$ model proposed by Yakhot and co-workers, excellent results for the backstep problem are now obtained. Interestingly enough, these results are not that sensitive to the details of the near wall treatment.

*Dedicated to A. C. Eringen on the occasion of his 70th birthday.

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1. INTRODUCTION

During the past two decades, the $K-\varepsilon$ model has played a central role in the calculation of many of the turbulent flows of scientific and engineering interest. In this two-equation model, the turbulence length and time scales are built up from the turbulent kinetic energy K and dissipation rate ε which are obtained from separate modeled transport equations. The Reynolds stress tensor is represented by an eddy viscosity model constructed from length and time scales based on K and ε. It was during the early 1970's that the original version of the $K-\varepsilon$ model was developed by Launder and co-workers [1, 2] based on a phenomenological approach which incorporated empirically many of the earlier ideas of Kolmogorov [3]. In the intervening years, a variety of modifications have been made to the $K-\varepsilon$ model to account for other complicating features such as near-wall turbulence, body forces and compressibility effects to name a few [4-7].

One of the major criticisms that has been leveled against the $K-\varepsilon$ model is that it is *ad hoc* - namely, it is not derived from the Navier-Stokes equations in a systematic fashion. However, Yakhot and Orszag [8] recently derived a version of the $K-\varepsilon$ model using Renormalization Group (RNG) methods. In this approach, an expansion is made about an equilibrium state with known Gaussian statistics by making use of the correspondence principle where the effects of mean strains are represented by a random force. Bands of high wavenumbers (namely, the small scales) are systematically removed and space is rescaled in a manner analogous to that employed in the study of phase transitions. The removal of only the smallest scales gives rise to subgrid scale models for large-eddy simulations whereas the removal of successively larger scales ultimately leads to Reynolds stress models. At high turbulence Reynolds numbers, the RNG based $K-\varepsilon$ model of Yakhot and Orszag [8] is of the same general form as the standard $K-\varepsilon$ model; however, the constants are calculated explicitly and assume somewhat different values. One major difference between the RNG and standard $K-\varepsilon$ model lies in the near wall treatment. The RNG model can be integrated directly to a solid boundary without the need for ad hoc wall damping functions. Several applications of this RNG $K-\varepsilon$ model have been subsequently reported by Yakhot, Orszag and co-workers. Probably the most notable one - since it deals with a complex shear flow with separation - is for turbulent flow over a backward facing step (see Karniadakis *et al.* [9]). It is well-known that the standard $K-\varepsilon$ model underpredicts the reattachment point in this problem - a deficiency that has been widely discussed in the literature since the 1980/81 Stanford Conference on Complex Turbulent Flows [10]. Karniadakis *et al.* [9] claimed that the RNG $K-\varepsilon$ model predicts reattachment 7.3 step heights downstream of the step corner for the Kim, Kline and Johnston [11] test case - a result that is extremely close to the experimental value. This apparent improvement in the predictions of the RNG based $K - \varepsilon$ model were attributed largely to the better treatment of near wall turbulence effects [9].

Subsequent to the publication of the paper by Karniadakis *et al.* [9], several adjustments have been made in the RNG $K - \varepsilon$ model. An error in the calculation of the constant $C_{\varepsilon 1}$ in the dissipation rate transport equation was corrected (see Smith and Reynolds [12] and Yakhot and Smith [13]). Furthermore, a double expansion technique was introduced by Yakhot *et al.* [14] in an effort to yield improved models for the production of dissipation term that can better accommodate large strain rates. In the latter paper, the backstep problem for the Kim, Kline and Johnston [11] test case was re-computed yielding excellent results for this latest version of the RNG $K - \varepsilon$ model. Consequently, we feel that there is a need to clarify several questions:

(1) Can the original version of the RNG $K - \varepsilon$ model yield good results for the backstep problem?

(2) Does the latest version of the RNG $K - \varepsilon$ model yield excellent results for the backstep problem due to the change in the constant $C_{\varepsilon 1}$, the addition of the new model for the production of dissipation term, or some other combination of factors?

While the first question was answered to the negative in a recent paper by Thangam and Speziale [15], there are still some open questions regarding the near wall modeling that need to be addressed. In the sections to follow, an effort will be made to fully clarify these issues and to gain a better understanding of the role of the various modeled terms in the prediction of turbulent separated flows.

2. THE RNG $K - \varepsilon$ MODEL AND THE BACKSTEP PROBLEM

The RNG procedure of Yakhot and Orszag [8] for incompressible turbulent flows yields the renormalized equations of motion

$$\frac{\partial \overline{u}_i}{\partial t} + \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} = -\frac{\partial \overline{P}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\nu + \nu_T)\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)\right] \tag{1}$$

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \tag{2}$$

where $\overline{u}_i$ is the mean velocity, $\overline{P}$ is the mean pressure, ν is the molecular viscosity and ν_T is the eddy viscosity. At high turbulence Reynolds numbers, the eddy viscosity takes the form

$$\nu_T = C_\mu \frac{K^2}{\varepsilon} \tag{3}$$

where

$$K = \frac{1}{2}\overline{u_i' u_i'}, \quad \varepsilon = \nu \overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}} \tag{4}$$

are, respectively, the turbulent kinetic energy and dissipation rate. C_μ is a dimensionless constant which was computed to be 0.085 using the RNG methodology. The turbulent kinetic energy and dissipation rate are typically obtained from modeled transport equations which, at high Reynolds numbers, take the form

$$\frac{\partial K}{\partial t} + \overline{u}_i \frac{\partial K}{\partial x_i} = \mathcal{P} - \varepsilon + \frac{\partial}{\partial x_i}\left(\frac{\nu_T}{\sigma_K}\frac{\partial K}{\partial x_i}\right) \tag{5}$$

$$\frac{\partial \varepsilon}{\partial t} + \overline{u}_i \frac{\partial \varepsilon}{\partial x_i} = C_{\varepsilon 1}\frac{\varepsilon}{K}\mathcal{P} - C_{\varepsilon 2}\frac{\varepsilon^2}{K} + \frac{\partial}{\partial x_i}\left(\frac{\nu_T}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right) \tag{6}$$

where $\mathcal{P}$ is the turbulence production given by

$$\mathcal{P} = 2\nu_T \overline{S}_{ij}\overline{S}_{ij} \tag{7}$$

and

$$\overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) \tag{8}$$

is the mean rate of strain tensor. The modeled transport equations (5) - (6) contain four coefficients: $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, σ_K and σ_ε. In the standard $K-\varepsilon$ model, these coefficients are constants which are obtained from benchmark experiments for equilibrium turbulent boundary layers and isotropic turbulence. On the other hand, in the original version of the RNG $K-\varepsilon$ model of Yakhot and Orszag [8], the coefficients are constants which are calculated explicitly by the theory. In the newest version of the RNG $K-\varepsilon$ model developed by Yakhot *et al.* [14], corrections to the constants in the dissipation rate equation made by Yakhot and Smith [13] were implemented along with a modification of the production of dissipation term to account for large strain rates. These coefficients can be summarized as follows:

Standard $K-\varepsilon$ Model [2]

$$\begin{gathered} C_\mu = 0.09, \ C_{\varepsilon 1} = 1.44, \ C_{\varepsilon 2} = 1.92 \\ \sigma_K = 1.0, \ \sigma_\varepsilon = 1.3 \end{gathered} \tag{9}$$

Original RNG $K-\varepsilon$ Model [8]

$$\begin{gathered} C_\mu = 0.085, \ C_{\varepsilon 1} = 1.063, \ C_{\varepsilon 2} = 1.72 \\ \sigma_K = 0.7179, \ \sigma_\varepsilon = 0.7179 \end{gathered} \tag{10}$$

New RNG $K-\varepsilon$ Model [14]

$$
\begin{aligned}
C_\mu = 0.085,\ C_{\varepsilon 1} &= 1.42 - \frac{\eta(1-\eta/\eta_0)}{1+\beta\eta^3} \\
C_{\varepsilon 2} = 1.68,\ \sigma_K &= 0.7179,\ \sigma_\varepsilon = 0.7179
\end{aligned}
\tag{11}
$$

where $\eta = SK/\varepsilon$, $S = (2\overline{S}_{ij}\overline{S}_{ij})^{1/2}$, $\eta_0 = 4.38$ and $\beta = 0.015$. The new RNG $K-\varepsilon$ model is obtained by implementing the corrections of Yakhot and Smith [13] (wherein $C_{\varepsilon 1}$ was changed from 1.063 to 1.42 and $C_{\varepsilon 2}$ from 1.72 to 1.68) along with a strain-dependent modification to $C_{\varepsilon 1}$ which is significant for flows with large strain rates. In so far as the latter modification is concerned, η_0 is the fixed point for homogeneously strained turbulent flows and β is a constant which was evaluated to yield a von Kármán constant $\kappa \approx 0.4$ (see Yakhot *et al.* [14]).

The test case to be considered is turbulent flow over a backward facing step (see Figure 1). Model predictions will be compared with the experimental data of Kim, Kline and Johnston [11] as updated by Eaton and Johnston [16] for the 1980/81 Stanford Conference on Complex Turbulent Flows. For this flow configuration, the expansion ratio E (step height: outlet channel height) is 1:3 and the Reynolds number Re is 132,000 based on the inlet centerline mean velocity and the outlet channel height.

The fully-developed mean velocity is of the two-dimensional form

$$
\overline{\mathbf{u}} = \overline{u}(x,y)\mathbf{i} + \overline{v}(x,y)\mathbf{j}. \tag{12}
$$

Hence, the governing field equations to be solved are given by

$$
\begin{aligned}
\overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} &= -\frac{\partial \overline{P}}{\partial x} + \frac{\partial}{\partial x}\left[2(\nu+\nu_T)\frac{\partial \overline{u}}{\partial x}\right] \\
&\quad + \frac{\partial}{\partial y}\left[(\nu+\nu_T)\left(\frac{\partial \overline{u}}{\partial y} + \frac{\partial \overline{v}}{\partial x}\right)\right]
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
\overline{u}\frac{\partial \overline{v}}{\partial x} + \overline{v}\frac{\partial \overline{v}}{\partial y} &= -\frac{\partial \overline{P}}{\partial y} + \frac{\partial}{\partial x}\left[(\nu+\nu_T)\left(\frac{\partial \overline{u}}{\partial y} + \frac{\partial \overline{v}}{\partial x}\right)\right] \\
&\quad + \frac{\partial}{\partial y}\left[2(\nu+\nu_T)\frac{\partial \overline{v}}{\partial y}\right]
\end{aligned}
\tag{14}
$$

$$
\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} = 0 \tag{15}
$$

$$\overline{u}\frac{\partial K}{\partial x}+\overline{v}\frac{\partial K}{\partial y} = \mathcal{P}-\varepsilon+\frac{\partial}{\partial x}\left[\left(\nu+\frac{\nu_T}{\sigma_K}\right)\frac{\partial K}{\partial x}\right] + \frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_T}{\sigma_K}\right)\frac{\partial K}{\partial y}\right] \tag{16}$$

$$\overline{u}\frac{\partial \varepsilon}{\partial x}+\overline{v}\frac{\partial \varepsilon}{\partial y} = C_{\varepsilon 1}\frac{\varepsilon}{K}\mathcal{P}-C_{\varepsilon 2}\frac{\varepsilon^2}{K}+\frac{\partial}{\partial x}\left[\left(\nu+\frac{\nu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x}\right] + \frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right] \tag{17}$$

where the turbulence production $\mathcal{P}$ takes the form

$$\mathcal{P}=2\nu_T\left(\frac{\partial \overline{u}}{\partial x}\right)^2+\nu_T\left(\frac{\partial \overline{u}}{\partial y}+\frac{\partial \overline{v}}{\partial x}\right)^2+2\nu_T\left(\frac{\partial \overline{v}}{\partial y}\right)^2 \tag{18}$$

given that the eddy viscosity ν_T is as defined in (3).

Equations (13) - (17) are solved subject to the following boundary conditions:

(a) The inlet mean velocity profile $\overline{u}$ is specified five step heights upstream of the step corner. This profile is matched to the experimental data [16] by performing a separate developing channel flow calculation which also provides companion profiles for K and ε at the inlet.

(b) Extrapolated outflow conditions are applied thirty step heights downstream of the step corner.

(c) The law of the wall is applied at the upper and lower walls in the three layer form [15]:

$$\overline{u}^+ = \begin{cases} y^+, & \text{for } y^+ \leq 5 \\ -3.05+5\ \ell n\ y^+, & \text{for } 5 < y^+ < 30 \\ 5.5+2.5\ \ell n\ y^+, & \text{for } y^+ \geq 30 \end{cases} \tag{19}$$

where $y^+ = yu_\tau/\nu$, $\overline{u}^+ = \overline{u}/u_\tau$ and u_τ is the friction velocity constructed from the wall shear stress in the standard manner. The law of the wall (18) is used in conjunction with the boundary condition of vanishing normal derivative of K at the wall along with the log layer formulas

$$\frac{K}{u_\tau^2}=C_\mu^{-1/2}, \quad \varepsilon = C_\mu^{3/4}\frac{K^{3/2}}{\kappa y} \tag{20}$$

(where $\kappa \approx 0.4$ is the von Kármán constant) which are suitably interpolated to establish boundary conditions for K and ε at the first grid point away from the wall [17]. The friction velocity u_τ is approximated by its log law form $u_\tau \approx C_\mu^{1/4} K^{1/2}$ in order to avoid problems with singularities at the reattachment point where the wall shear stress vanishes.

The equations of motion are solved in their time dependent form by a finite volume method. A variable 200×100 mesh, which concentrates mesh points near the step corner, is used. This resolution is required to ensure the numerical accuracy of the results (many earlier computations of the backstep problem were under-resolved; see Thangam and Speziale [15] for a discussion of this issue as well as for other details of the numerical method). The computations were conducted on a CRAY-YMP computer. Approximately 15 minutes of CPU time is required to obtain a steady state solution for a given test case.

3. DISCUSSION OF THE RESULTS

First, we will present computed results for the standard $K - \varepsilon$ model since this is an important benchmark against which the other results will be judged. All model predictions will be compared with experimental data for the Kim, Kline and Johnston test case as updated by Eaton and Johnston [16] for the 1980/81 Stanford Conference on Complex Turbulent Flows. In Figures 2(a)-(b) the mean velocity streamlines and mean velocity profiles obtained from the standard $K - \varepsilon$ model are compared with the experimental data. The standard $K - \varepsilon$ model predicts reattachment at $X_R/H \approx 6.25$ - a value that constitutes approximately a 12% underprediction of the mean experimental reattachment point of $X_R/H \approx 7.1$. Earlier reported results [10] of a 20-25% underprediction of the reattachment point appear now to have been in error due to insufficient numerical resolution as discussed by Thangam and Speziale [15] and Avva, Kline and Ferziger [18].

One of the criticisms that has been leveled against most of the $K - \varepsilon$ model calculations for the backstep problem lies in the use of wall functions. It is well-known that wall functions do not formally apply to separated flows. However, since the separation point is fixed - and, perhaps, since a large proportion of the turbulent kinetic energy is not associated with the separated zone - it now appears that wall functions do not give rise to major errors in the backstep problem, unlike in other separated flows. A systematic study of near wall turbulence models by the authors has indicated that they generally do not give rise to more than a 5% improvement in the results for the backstep problem obtained from *well-resolved* computations using wall functions. In Table 1, the reattachment point prediction of the standard $K - \varepsilon$ model using three-layer wall functions is compared with experimental data [16] as well as with the result obtained using the asymptotically consistent near wall model of Speziale, Abid and Anderson [19]. Here, this particular near wall model is chosen since it was shown by those authors to be better behaved than four independent near-wall turbulence models that were recently reviewed by Patel, Rodi and Scheuerer [20] for boundary layer flows. It is clear from Table 1 that the use of an asymptotically consistent near wall model does not lead to a significant improvement in the results obtained using wall functions. Furthermore, it should be noted that the use of an asymptotically inconsistent near wall model (some of which are discussed in Patel, Rodi and Scheuerer [20]) can lead to even *worse* results than those obtained using wall functions! Consequently, we believe that it is acceptable to use wall functions when comparing the predictions of a variety of turbulence models in the backstep problem.

Now we will present computed results for the original version of the RNG $K-\varepsilon$ model (Yakhot and Orszag [8]). In Figures 3(a) - (b) the computed mean velocity streamlines and mean velocity profiles are compared with the experimental data of Eaton and Johnston [16]. The surprising finding is that the predicted mean reattachment point of $X_R/H \approx 4.0$ constitutes over a 40% underprediction of the mean experimental result of $X_R/H \approx 7.1$. This deficiency - as well as a plausible explanation for its cause - was pointed out recently by Thangam and Speziale [15]. In a homogeneous shear flow, the eddy viscosity grows exponentially:

$$\nu_T \sim \exp(\lambda t^*)$$

where t^* is the time (nondimensionalized by the shear rate) and λ is the growth rate given by (see Speziale [21])

$$\lambda = \left[\frac{C_\mu(C_{\varepsilon 2} - C_{\varepsilon 1})^2}{(C_{\varepsilon 1} - 1)(C_{\varepsilon 2} - 1)}\right]^{1/2} \tag{21}$$

which becomes singular for $C_{\varepsilon 1} = 1$. Consequently, for $C_{\varepsilon 1} = 1.063$, the growth rate of the eddy viscosity will be overly large. When a model overpredicts the eddy viscosity it will be too dissipative - a deficiency that will lead to an underprediction of the separated flow region in the backstep problem. Earlier reported results by Karniadakis *et al.* [9] for the backstep problem - which suggested that the model predictions were excellent - now appear to have been in error.

With the correction of $C_{\varepsilon 1}$ to 1.42 by Yakhot and Smith [13], the problem of overpredicting the growth rate of homogeneous shear flow is eliminated. Speziale, Gatski and Fitzmaurice [22] showed that with the new value of $C_{\varepsilon 1}$, the RNG $K-\varepsilon$ model yields excellent results for homogeneous shear flow. However, an additional production term in the dissipation rate transport equation was uncovered which Yakhot and Smith [13] were not able to close systematically. The new term was shown to be important for large strain rates. If this term is neglected, a revised RNG $K-\varepsilon$ model is obtained which is of the same general form as the Yakhot and Orszag [8] model; only the values of the constants are altered. In Figures 4(a)-(b), the mean velocity streamlines and mean velocity profiles obtained from the Yakhot-Smith RNG $K - \varepsilon$ model (ignoring the additional production term) are shown. It is clear that the model drastically overpredicts the reattachment point (i.e., $X_R/H \approx 9.7$ in comparison to the mean experimental result of 7.1). The origin of this erroneous prediction is clear: with the additional production term in the ε-transport equation neglected, the Yakhot-Smith model yields a von Kármán constant of 0.23 instead of the traditional value of 0.4 (see Yakhot and Smith [13]). This strongly indicates that the additional production term cannot be neglected in wall bounded turbulent flows.

Yakhot *et al.* [14] recently developed a model for the additional production term in the ε-transport equation by means of a scale expansion. More precisely, an expansion of this term was made in the ratio of the turbulent to mean strain time scale $\eta \equiv SK/\varepsilon$, where $S = (2\overline{S}_{ij}\overline{S}_{ij})^{1/2}$ is the norm of the mean rate of strain tensor. The interesting finding was that no finite truncation of the expansion for the production of dissipation term in powers of η suffices - terms of all orders must be retained to satisfy the crucial weak and strong strain limits [14]. This complication eliminated the possibility of obtaining a closed form solution for this term. A Padé approximation was made which had one undetermined constant β that was evaluated empirically by setting the von Kármán constant to 0.4. This leads to a net production of dissipation term of the traditional form $C_{\varepsilon 1}(\varepsilon/K)\mathcal{P}$ with a variable coefficient given by

$$C_{\varepsilon 1} = 1.42 - \frac{\eta(1 - \eta/\eta_0)}{1 + \beta\eta^3} \tag{22}$$

where $\beta = 0.015$ and $\eta_0 = 4.38$ is the fixed point for equilibrium homogeneous flows. The addition of the second term to the r.h.s. of (22) - which becomes significant for large strain rates $\eta > \eta_0$ - is the feature that distinguishes the Yakhot *et al.* [14] model from the Yakhot and Smith [13] model.

In Figures 5(a)-(b), the computed mean velocity streamlines and mean velocity profiles for the Yakhot *et al.* [14] RNG $K - \varepsilon$ model are compared with experimental data [16]. The model predicts reattachment at $X_R/H \approx 6.7$: a result that is within 5% of the experimental mean reattachment point of 7.1. Most notably, this new RNG $K - \varepsilon$ model - unlike all of the other models - yields a significant secondary separation bubble below the corner of the step that is more in line with the experimental data [23]. Yakhot *et al.* [14] showed that when this model is extended to include an anisotropic eddy viscosity, the predicted mean reattachment point is almost identical to the experimental value of $X_R/H \approx 7.1$.

4. CONCLUDING REMARKS

Three independent versions of the RNG $K-\varepsilon$ model have been applied to the test case of turbulent flow over a backward facing step in order to assess its performance in separated flows. The main objective of this study was to better understand the impact that the recent changes in the Yakhot & Orszag RNG $K-\varepsilon$ model have had on the predictive capabilities of the model – particularly in a complex shear flow of engineering interest. It was found that the original version of the RNG $K-\varepsilon$ model substantially underpredicts the reattachment point for the Kim, Kline and Johnston test case due to an overprediction of the eddy viscosity (the model is far too dissipative). However, the latest version of the RNG $K-\varepsilon$ model [14] – which is an extension of the Yakhot and Smith [13] model to accommodate large strain rates in the production of dissipation term – yields excellent results for the backstep problem. The mean reattachment point is predicted to within 5% of the experimental value in comparison to the standard $K-\varepsilon$ model which yields a 12% error. Furthermore, a secondary separation bubble was obtained that is much more in agreement with that which is observed in experiments. These results are strongly dependent on the presence of the variable strain rate term in (21) and the calculations vividly demonstrate the sensitivity of the reattachment point to the values of $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$. The encouraging point, however, is that the results are not that sensitive to the details of the near wall treatment and are obtained with no *ad hoc* adjustments of the constants. Based on these results, we feel that the current version of the RNG $K-\varepsilon$ model can be a useful turbulence model for practical engineering and scientific calculations.

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NEAR-WALL MODEL	X_R/H
Three-layer wall functions	6.25
Speziale *et al.* [19]	6.40
Experimental data [16]	7.1

Table 1. The reattachment point in the backstep problem: Comparison of the predictions of different near wall corrections to the standard $K-\varepsilon$ model with experimental data.

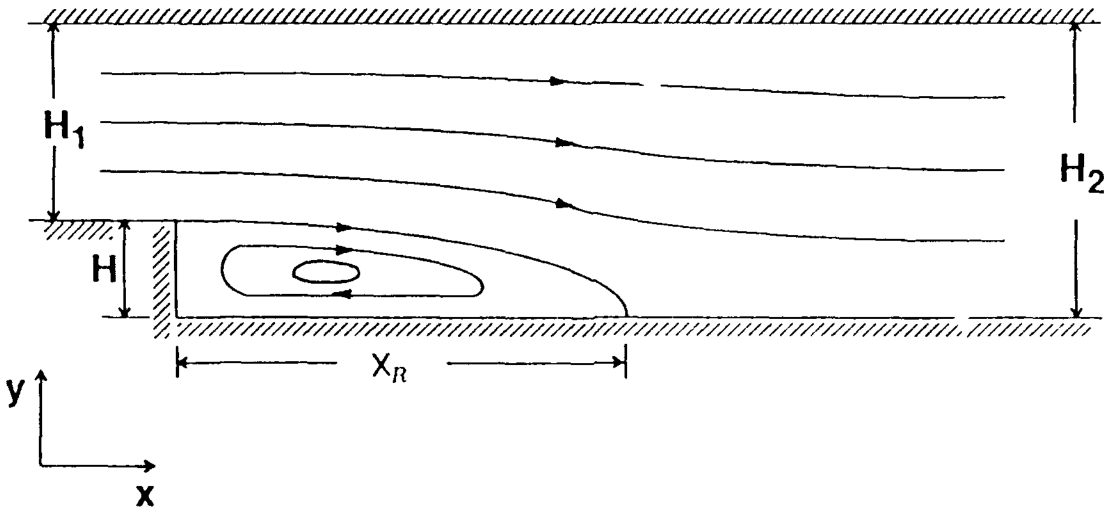


Figure 1. Schematic of turbulent flow over a backward facing step.

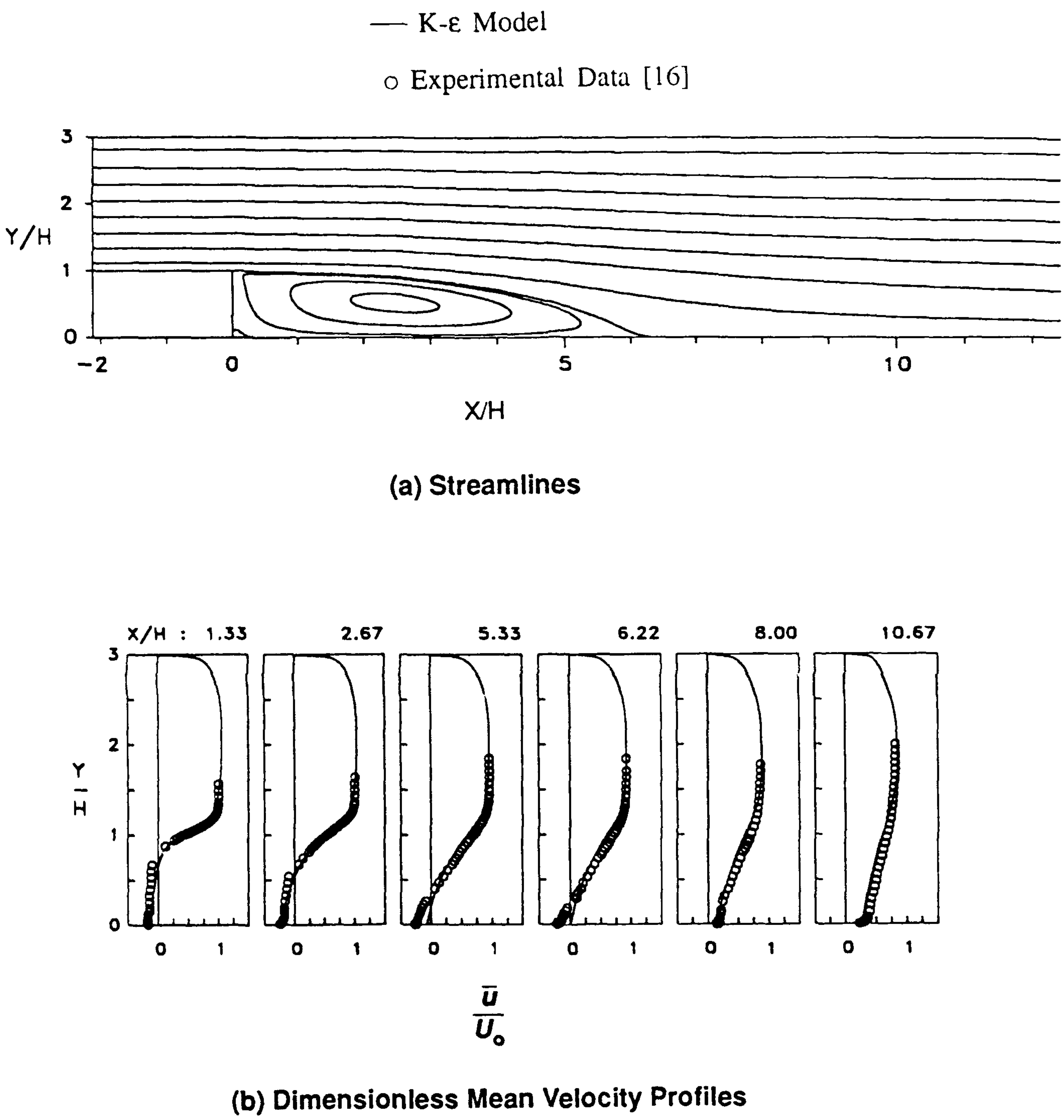
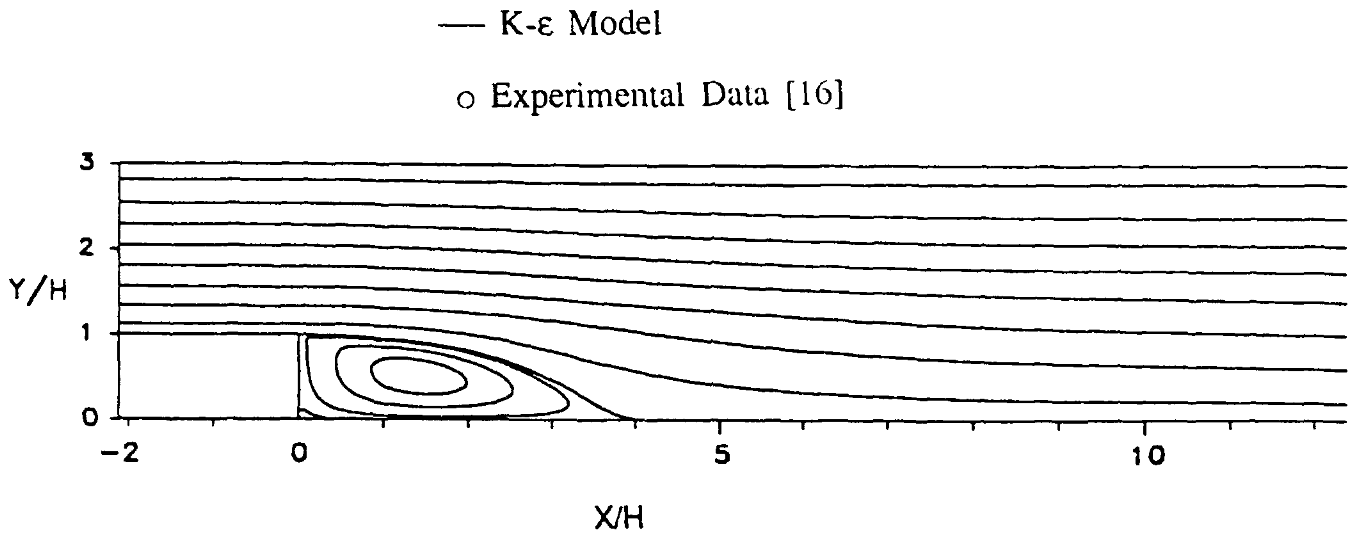
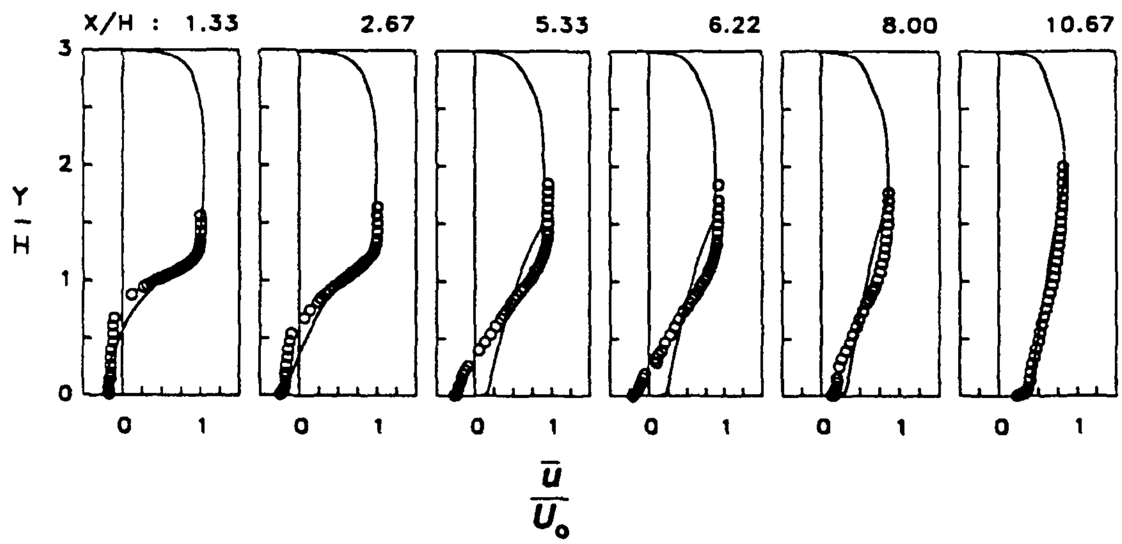


Figure 2. Comparison of the predictions of the standard $K-\varepsilon$ model [2] with experimental data [16]. (a) Streamlines and (b) mean velocity profiles.

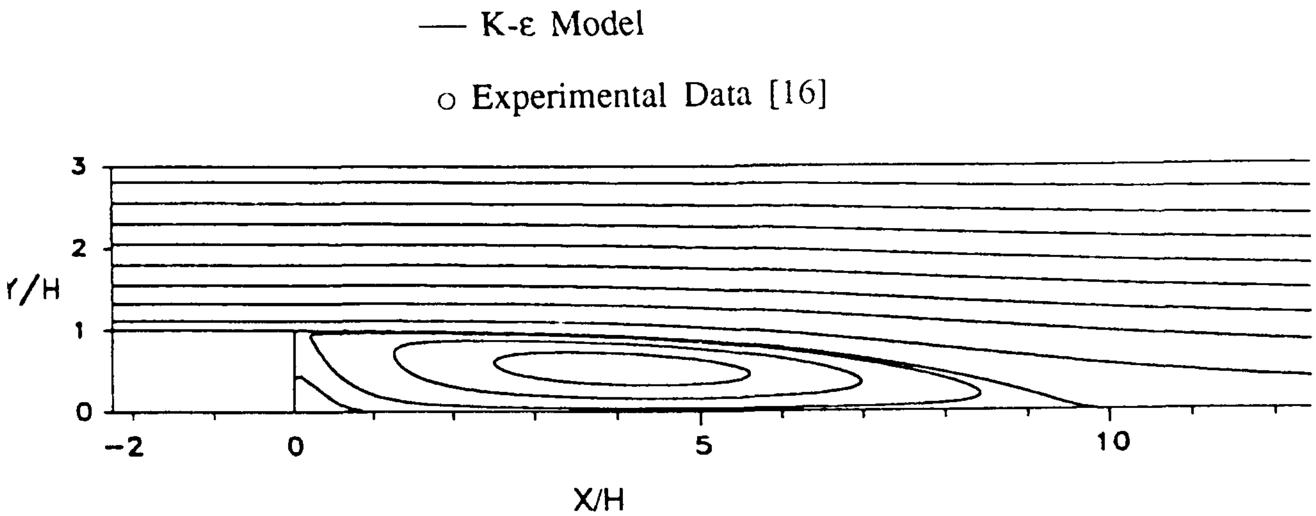


(a) Streamlines

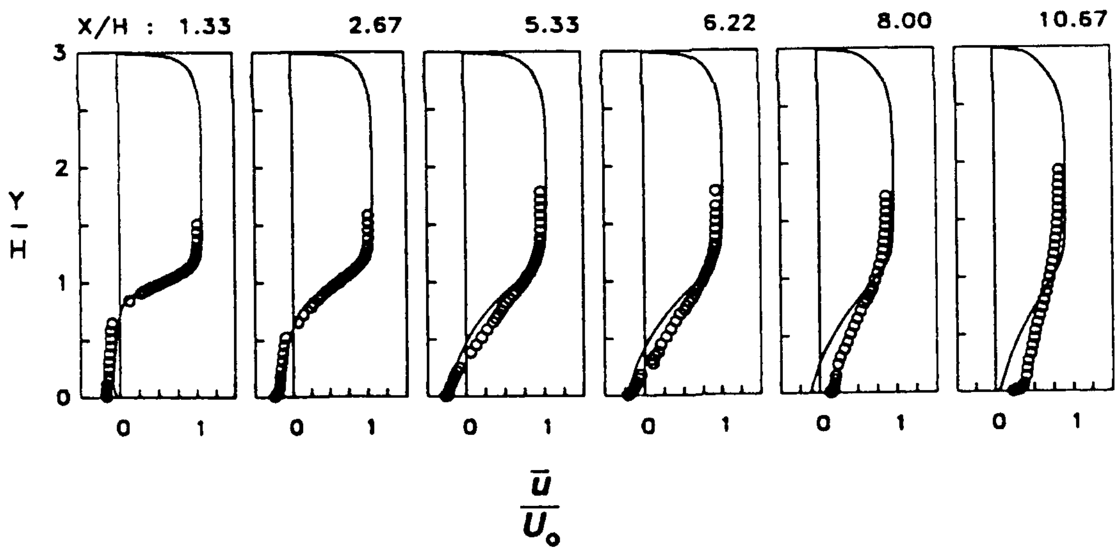


(b) Dimensionless Mean Velocity Profiles

Figure 3. Comparison of the predictions of the RNG $K-\varepsilon$ model of Yakhot and Orszag [8] with experimental data [16]. (a) Streamlines and (b) mean velocity profiles.

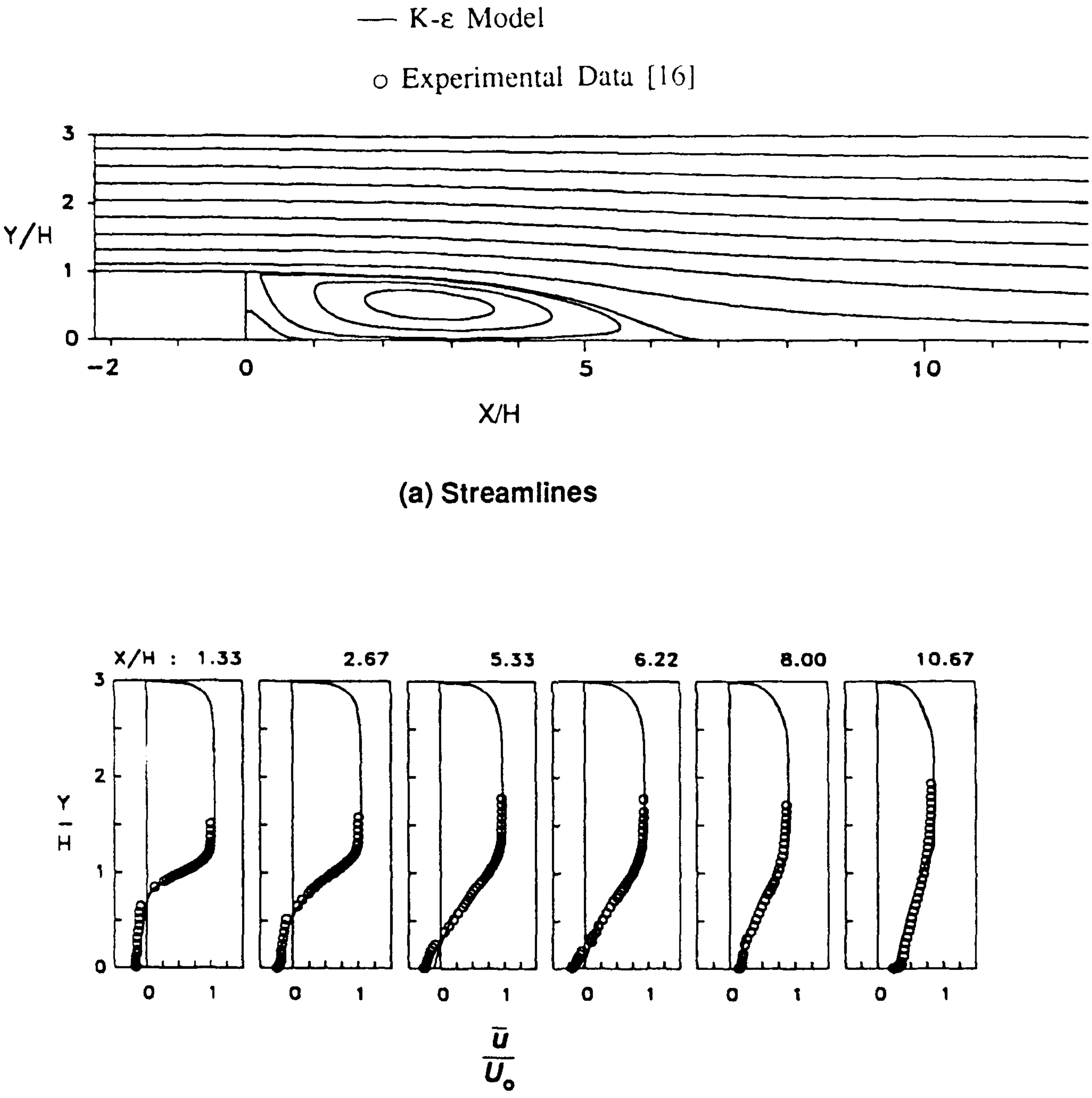


(a) Streamlines



(b) Dimensionless Mean Velocity Profiles

Figure 4. Comparison of the predictions of the RNG $K-\varepsilon$ model of Yakhot and Smith [13] - where the additional production of dissipation term is neglected - with experimental data [16]. (a) Streamlines and (b) mean velocity profiles.



(a) Streamlines

(b) Dimensionless Mean Velocity Profiles

Figure 5. Comparison of the predictions of the RNG $K - \varepsilon$ model of Yakhot *et al.* [14] with experimental data [16]. (a) Streamlines and (b) mean velocity profiles.

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13. ABSTRACT *(Maximum 200 words)*

A two-equation turbulence model of the K-ε type was recently derived by Yakhot & Orszag based on Renormalization Group (RNG) methods. It was later reported that this RNG based model yields substantially better predictions than the standard K-ε model for turbulent flow over a backward facing step -- a standard test case used to benchmark the performance of turbulence models in separated flows. The apparent improvements obtained from the RNG K-ε model were attributed to the better treatment of near wall turbulence effects. In contrast to these earlier claims, it is shown in this paper that the original version of the RNG K-ε model substantially underpredicts the reattachment point in the backstep problem -- a deficiency that is traced to the modeling of the production of dissipation term. However, with the most recent improvements in the RNG K-ε model proposed by Yakhot and co-workers, excellent results for the backstep problem are now obtained. Interestingly enough, these results are not that sensitive to the details of the near wall treatment.

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